Quiz 3 solutions and marking scheme.

1. a) We need to show that $E\left(\frac{1}{X}\right) > \frac{1}{E(X)}$ for any positive non constant r.v. X.

The function $g(x) = \frac{1}{x}$ is strictly convex because
- 2 marks.

$g''(x) = \frac{2}{x^3} > 0 \quad \forall\, x > 0.$ - 1 mark.

So by Jensen's inequality, we have

$$E\left(\frac{1}{X}\right) > \frac{1}{E(X)} \qquad \left[E(g(X)) > g(E(X)) \right].$$

for any positive non constant r.v. X.

- 2 marks.

b). $X, Y > 0$ r.v.s , with neither a constant multiple of another, we need to show that

$$E\left(\frac{X}{Y}\right) \cdot E\left(\frac{Y}{X}\right) > 1.$$

Let us define $W = \frac{X}{Y} > 0$ 3 marks.

Then by a) we have

$$E\left(\frac{1}{W}\right) > \frac{1}{E(W)} . \quad (*)$$

Now, $E\left(\frac{1}{W}\right) = E\left(\frac{Y}{X}\right)$ and $E(W) = E\left(\frac{X}{Y}\right)$

so using these in $(*)$, we have

$$E\left(\frac{Y}{X}\right) \cdot E\left(\frac{X}{Y}\right) > 1 .$$ Hence, proved.

— 2 marks.